SCHOOL

Trial WACE Examination, 2011

Question/Answer Booklet

MATHEMATICS 3A/3B(1)

Section Two: Calculator-assumed

SOLUTIONS

Student Number: In figures | | | | | | | | |

In words MARKING KEY

Your name _____

Time allowed for this section

Reading time before commencing work: ten minutes
Working time for this section: one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet
Formula Sheet (retained from Section One)

To be provided by the candidate

Standard items: pens, pencils, pencil sharpener, eraser, correction fluid/tape, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators satisfying the conditions set by the Curriculum Council for this examination.

Important note to candidates

No other items may be used in this section of the examination. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	7	7	50	40	33
Section Two: Calculator-assumed	13	13	100	80	67
			Total	120	100

Instructions to candidates

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2011*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in the spaces provided in this Question/Answer Booklet. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.

3. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

4. It is recommended that you **do not use pencil**, except in diagrams.

See next page

Section Two: Calculator-assumed **(80 Marks)**

This section has **thirteen (13)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 100 minutes.

Question 8 **(3 marks)**

The scores of 25 students in a test out of 70 are listed below in ascending order.

25	26	28	28	29
34	36	39	44	45
47	48	51	54	55
55	57	58	60	60
60	61	64	68	69

(a) How many students scored the same as the mode? (1 mark)

3 students ✓

(Mode is 60)

(b) Calculate the interquartile range for these scores. (2 marks)

Q3 = 60 ✓
Q1 = 35

IQR = 25 ✓

Question 9 **(9 marks)**

The length of time between calls of the grey tree frog is known to vary with the ambient temperature. Data for temperatures from 11°C to 33°C are shown in the table below.

Temperature (°C)	x	11	12	14	15	17	18	20	21	23	24	26	30	32	33
Time between calls (seconds)	y	4.3	5.1	5.9	6	7.1	6.9	5.8	6.1	5.2	5.2	4.4	3.5	2.5	2.4

(a) Use your calculator to graph the data and comment on any trends you notice. (2 marks)

The time between calls appears to increase for temperatures from 11°C to 17°C and then decrease linearly from 17°C to 33°C. ✓ ✓

(b) By careful selection of data from the above table, calculate a linear regression model of the form $y = mx + c$ suitable for temperatures above 16°C. (2 marks)

Use 10 data points in table with temperatures above 16°C

$y = -0.292x + 12.041$ ✓ ✓

(c) How confident are you that a linear model is appropriate in part (b)? Justify your answer with reference to Pearson's correlation coefficient. (2 marks)

The very strong negative correlation coefficient of -0.993 ✓ ✓ together with a visual inspection of the line indicates that we can be confident in the choice of a linear model.

(d) Calculate the residual time for a temperature of 20°C using the regression model from part (b). (2 marks)

$-0.292(20) + 12.041 = 6.196$ ✓

$5.8 - 6.196 = -0.396 \approx -0.4$ seconds ✓

(e) Comment on the statement "Due to the strong association evident between temperature and time, we can be certain that high temperatures cause grey tree frogs to have shorter time intervals between calls." (1 mark)

The statement is false ✓ in that we cannot imply causality from correlation. We can only observe that a relationship exists between the variables.

Must give a comment to justify/elaborate false.

Question 10 **(4 marks)**

The two variables h and w are inversely proportional to one another.

(a) Circle each of the equations below that reflect this relationship, where k is a constant. (2 marks)

$h + w = k$ $w = hk$ $wh = k$ ✓ $\frac{h}{k} = w$ $\frac{k}{w} = h$ ✓ $\frac{h}{w} = k$

−1 for each extra circle

(b) When $h = 12.5$, $w = 38.8$. If h decreases by 2.8, by how much will w change? (2 marks)

$12.5 - 2.8 = 9.7$

$12.5 \times 38.8 = 9.7 \times w$ ✓ f/t from (a)

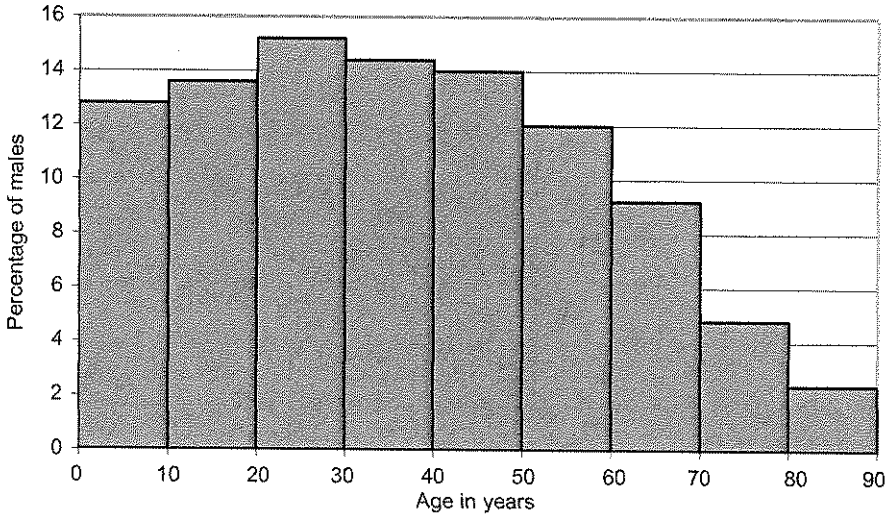
$w = 50$

$50 - 38.8 = 11.2$ ✓

w will increase by 11.2

Question 11 **(9 marks)**

The histogram below shows the distribution of ages of the under-90 male population of Australia in the year 2010. The mean and standard deviation of the data shown are 36.6 and 21.7 years respectively.



(a) Describe the spread of the data shown in the histogram. (2 marks)

The modal age is 20 to 30 years, with just over 15% of males in this group.

The percentage in each group increase slightly from 0 to 20 years and then decrease slightly from 30 to 60 years.

After 60 years, the percentages drop significantly, as is evident in the positive skewness shown in the histogram.

✓ ✓ 1 mark for each reasonable statement

(b) If the data had been presented as a boxplot, would you expect the length of the left hand whisker to be greater than, about the same as, or less than, the length of the right hand whisker? Explain your answer. (2 marks)

LH whisker will be less than the length of the RH whisker, due to positive skew of data. ✓

Also observe from the histogram that about 26% of data lies in interval 0 to 20 years yet only 16% of data lies in the larger interval 60 to 90 years. ✓

(for less 1m, reason 1m)

The table below shows the distribution of ages of the under-90 female population of Australia in the year 2010.

Age in years (x)	Percentage of females
$0 \le x < 10$	12.3
$10 \le x < 20$	12.7
$20 \le x < 30$	14.6
$30 \le x < 40$	14.8
$40 \le x < 50$	14.2
$50 \le x < 60$	12.6
$60 \le x < 70$	9.4
$70 \le x < 80$	5.7
$80 \le x < 90$	3.7

(c) Calculate the mean and standard deviation of the female ages. (2 marks)

Mean is 38.2 years. ✓

Standard deviation is 22.3 years. ✓

(d) Comment on any similarities or differences between the male and female age distributions. (3 marks)

The mean male age is slightly lower than that of the females (36.6 compared to 38.2 years), indicating that females tend to live longer. This is also evident when comparing the percentages in the 70 to 90 age groups (~7% compared to 9.4%). ✓

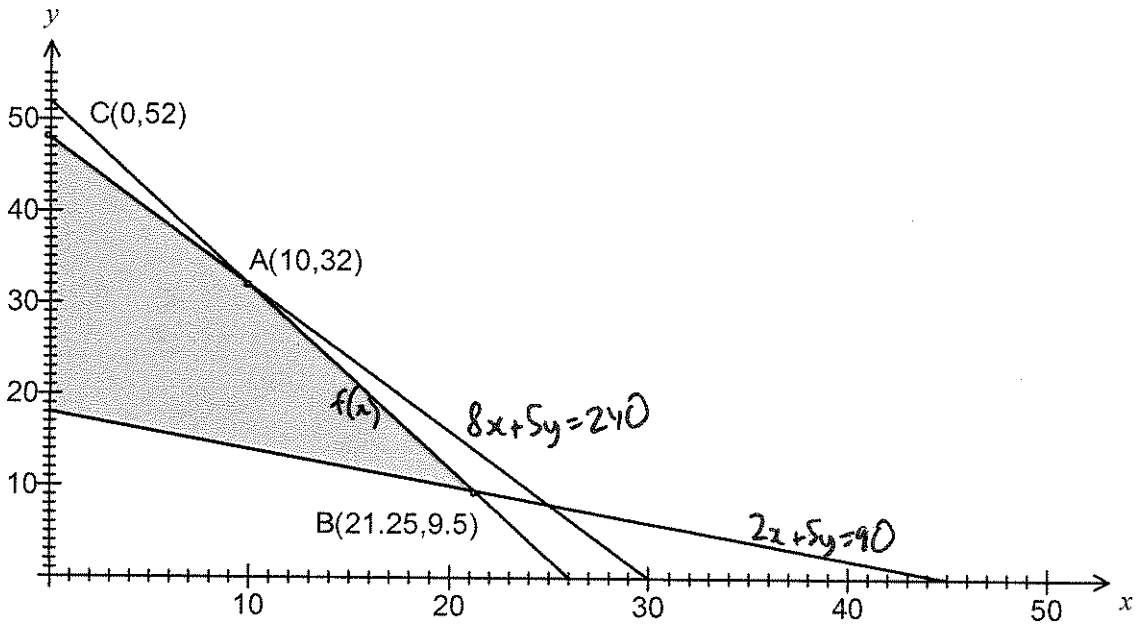
The female ages are very slightly more spread than those of males when comparing the standard deviations (22.3 compared to 21.7 years). ✓

Both distributions show positive skew, as the percentages in the older age groups decrease. ✓

1 mark for each reasonable statement

Question 12 **(8 marks)**

The graph below shows the lines $2x+5y=90$ and $y=f(x)$.



(a) Determine the equation of $f(x)$. (1 mark)

$y=-2x+52$ ✓

(b) Add the line $8x+5y=240$ to the graph above. ✓ (1 mark)

(c) Shade the feasible region defined by $x\geq 0$, $y\geq 0$, $8x+5y\leq 240$, $2x+5y\geq 90$ and $y\leq f(x)$. ✓✓ −1 each error, but f/t (2 marks)

(d) Find the values of x and y that maximise the objective function $15x+7y$ subject to the constraints in part (c) and state this maximum value. Justify your answer. (4 marks)

$15(0)+7(52)=364$
$15(10)+7(32)=374$ ✓✓
$15(21.25)+7(9.5)=385.25$
$\therefore x=21.25, y=9.5$ for maximum value of 385.25 ✓ ✓

Question 13 **(6 marks)**

A quality control officer at a soft-drink bottling plant uses systematic sampling to select close to 0.2% of bottles filled for checking.

(a) Describe a practical way that the systematic sampling might be carried out. (2 marks)

$0.2\% = \frac{1}{500}$

Pick the first bottle at random from the production line and then remove every 500th bottle after that.

✓✓ or other reasonable method.
Must have 1 in 500 ratio for full marks

On a particular day, the content of a bottle of soft-drink was observed to be normally distributed with a mean of 382.3 mL and a standard deviation of 2.9 mL.

(b) How many bottles in a sample of 500 would be expected to contain less than the stated content of 375mL? (2 marks)

$P(X < 375) = 0.0059$ ✓

$500 \times .0059 \approx 3$ bottles ✓

full marks for answer only

(c) Calculate the 95th percentile for this distribution. (2 marks)

$P(X < k) = 0.95$
$k = 387.1$ mL

± 0.1 ✓✓ full marks for answer only

Question 14 **(4 marks)**

Consider the function $f(x) = x^3 + 6x^2 + 12x + 8$.

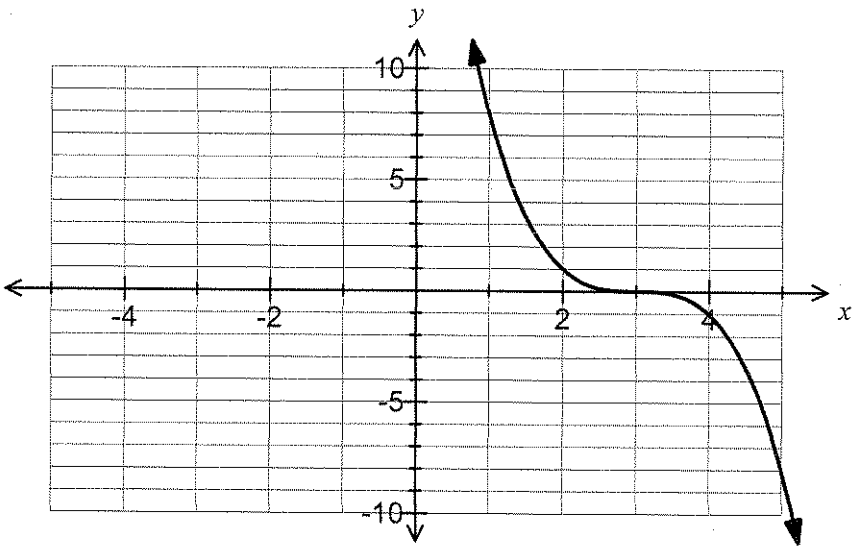
(a) Use your calculator to factorise $f(x)$. (1 mark)

$f(x) = (x+2)^3$ ✓

(b) Describe how the graph of $y = f(x)$ can be obtained from the graph of $y = x^3$. (1 mark)

Translate the graph of $y = x^3$
2 units to the left. ✓

(c) The graph of the function $y = g(x)$ is shown below and is congruent with the graph of $y = f(x)$. Determine $g(x)$ in the expanded form $ax^3 + bx^2 + cx + d$. (2 marks)



$g(x) = -(x-3)^3$ ✓
$= -x^3 + 9x^2 - 27x + 27$ ✓

-1 if not expanded

Question 15 **(4 marks)**

A thin piece of glass has been cut into the shape of an obtuse-angled triangle with an area of 135.5 cm^2 and two sides of 21.8 cm and 25.4 cm. Calculate the length of the third side, correct to 3 significant figures.

$135.5 = 0.5 \times 21.8 \times 25.4 \times \sin\theta$ ✓

$\therefore \theta = 150.7°$ (obtuse solution) ✓

$x^2 = 21.8^2 + 25.4^2 - 2 \times 21.8 \times 25.4 \times \cos 150.7°$ ✓ or sketch of triangle using geometry mode from classpad.

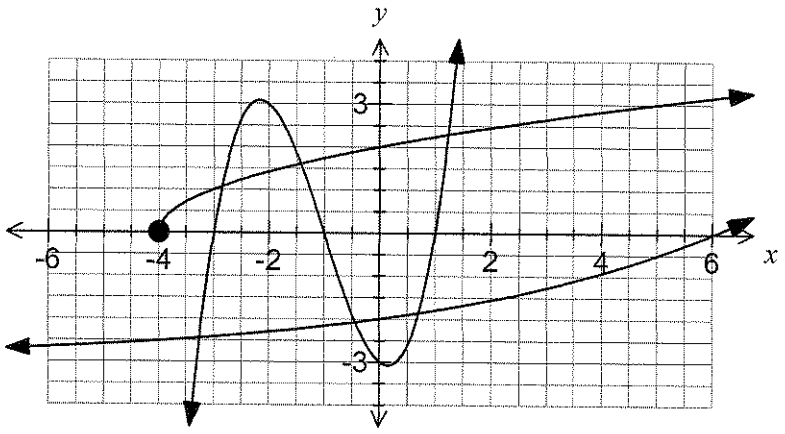
$x = 45.67$

$x \approx 45.7$ cm to 3sf ✓

−1 if not 3sf

Question 16 **(7 marks)**

Three functions are given by $f(x)=1.2^x-3$, $g(x)=\sqrt{x+4}$ and $h(x)=x^3+3x^2-x-3$.
Their graphs are shown on the axes below.



(a) What is the domain of $g(x)$? (1 mark)

$\{x : x \geq -4\}$ ✓

(b) For which function is the domain the same as the range? (1 mark)

$h(x)$ ✓

(c) State which function has an asymptote and write down the equation of this asymptote. (2 marks)

$f(x)$ ✓

$y=-3$ ✓

(d) Use the graph to estimate all solutions to $g(x)=h(x)$. (2 marks)

$x=-2.8,\ -1.4,\ 1.2$ approx ±0·1

✓✓ −1 each missing

(e) From the above graph it could be deduced that there are 3 solutions to the equation $1.2^x-3=x^3+3x^2-x-3$. Briefly explain whether or not you agree that there are 3 solutions to the equation. (1 mark)

No. The two functions can be seen to intersect on the graph in 3 places, but a fourth solution exists at $x=70.18$ ✓

Question 17 **(5 marks)**

The daily sales figures for a new clothing store are shown in the table below. The figures in column a are the six-point centred moving averages of the sales data, rounded to the nearest dollar. The regression line of a on t from this data is $a = 122.5t + 2871$, with a correlation coefficient of 0.97.

Date	t	Sales ($)	a
Thu 05 May	1	3700	
Fri 06 May	2	2730	
Sat 07 May	3	2850	
Mon 09 May	4	4190	3353
Tue 10 May	5	1950	3496
Wed 11 May	6	4280	3605
Thu 12 May	7	4530	3723
Fri 13 May	8	3620	3859
Sat 14 May	9	3270	3973
Mon 16 May	10	5180	4083
Tue 17 May	11	2600	4223
Wed 18 May	12	5000	4343
Thu 19 May	13	5120	
Fri 20 May	14	4710	
Sat 21 May	15	3630	

(a) Write down the calculation used to find the six-point centred moving average for Mon 16 May. (1 mark)

$$\frac{4530 \div 2 + 3620 + 3270 + 5180 + 2600 + 5000 + 5120 \div 2}{6}$$ ✓

(b) Predict, with seasonal adjustment, the likely sales for Mon 23 May, assuming that existing trends continue. (4 marks)

Calculate Mon residuals and seasonal component

$4190 - 3353 = 837$

$5180 - 4083 = 1097$

$\frac{837 + 1097}{2} = 967$ ✓

Use trend line for sales when t=16 ✓

$122.5(16) + 2871 = 4831$ ✓ flt

Adjust for season to get prediction

$4831 + 967 = \$5798$ ✓

$\approx \$5800$

Question 18 **(7 marks)**

The spreadsheet below was used to model the balance of an annuity used to help pay the university fees of a student. \$19 000 was deposited in a fixed-interest savings account at the start of the year 2011. One year later interest was added and then the student withdrew \$4 480. This process was repeated until the balance of the annuity reached zero at the start of the year 2016.

Date	Balance	Interest	Withdrawal
1/01/2011	19000.00	1092.50	4480.00
1/01/2012	15612.50	897.72	4480.00
1/01/2013	12030.22	691.74	4480.00
1/01/2014	8241.96	473.91	4480.00
1/01/2015	4235.87	243.56	4479.43
1/01/2016	0.00		

(a) How much did the annuity provide in total towards the student's university fees? (1 mark)

$4480 \times 4 + 4479.43 = \$22\,399.43$ ✓

(b) What was the annual percentage interest rate? (1 mark)

$1092.50 \div 19000 \times 100 = 5.75\%$ ✓

(c) Explain why the final withdrawal was less than \$4 480. (1 mark)

Because the previous balance plus interest only amounted to \$4 479.43 at the start of 2016 and so there was not enough to pay \$4 480. ✓

(d) Write a recursive rule to generate the balance at the start of each year from 2011 until 2015. (2 marks)

$T_{n+1} = T_n \times 1.0575 - 4480 \quad T_1 = 19000$ ✓ ✓

(e) Suggest two changes to this annuity so that it would provide a larger amount towards the student's university fees. (2 marks)

Increase the initial deposit. ✓
Increase the annual interest rate. ✓
Compound the interest more frequently.

1 for each reasonable method

Question 19 **(6 marks)**

A cylinder of radius r and height h is such that the sum of the radius and height is 42cm.

(a) Show that the volume of the cylinder is given by $V = 42\pi r^2 - \pi r^3$. (2 marks)

$r + h = 42$

$\therefore h = 42 - r$ ✓

$V = \pi r^2 h$

$= \pi r^2 (42 - r)$

$= 42\pi r^2 - \pi r^3$ ✓

(b) Use calculus techniques to find the maximum possible volume of the cylinder, correct to 3 significant figures. (4 marks)

$\frac{dV}{dr} = 84\pi r - 3\pi r^2$ ✓

$= 0$ when $r = 0$ or 28

$\therefore$ maximum volume when r=28 ✓

$V = 34482$

or

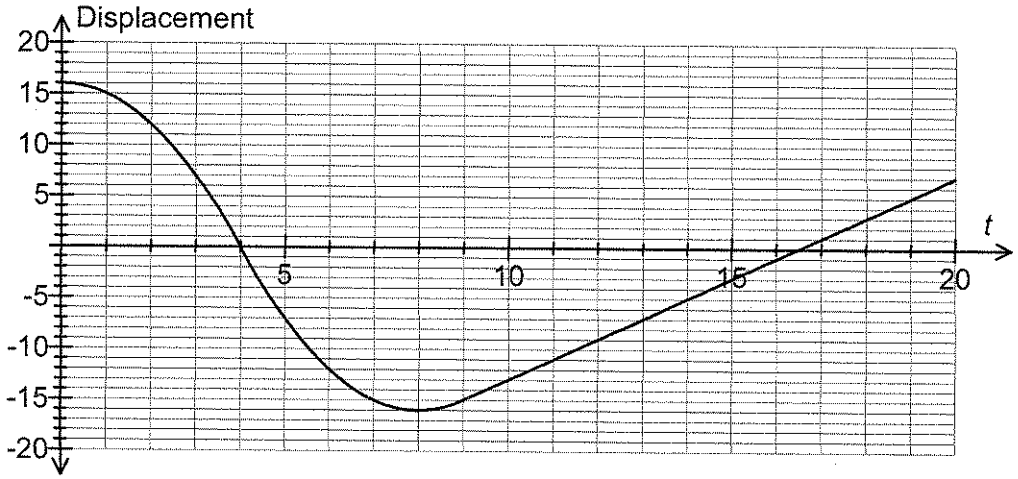
≈ 34500 cm^3 t[redacted] ✓

✓ sign test or $\frac{d^2y}{dx^2}$ or sketch to show local max

DO NOT penalise for incorrect rounding

Question 20 **(8 marks)**

A small toy train is able to travel backwards and forwards along a straight track built on level ground. The displacement in metres, of the train relative to point A, is shown on the graph below for the interval $0 \le t \le 20$ seconds.



(a) State an interval of time during which the train is moving towards point A. (1 mark)

$0 < t < 4$ or $9 < t < 16.5$ ✓

(b) What total distance did the train travel during the 20 second interval? (1 mark)

$16 + 16 + 16 + 7 = 55$ metres ✓

(c) In what interval of time was the rate of change of displacement constant and what was that rate of change, in centimetres per minute? (3 marks)

$9 \le t \le 20$ seconds ✓

Gradient $= 2$ m/s ✓

$= 2 \times 100 \times 60$

$= 12000$ cm/min ✓

(d) Estimate the area between the time axis and the displacement graph for the interval $0 \le t \le 4$ seconds using the average area of circumscribed and inscribed rectangles with widths of 1 second. (3 marks)

Circumscribed: 16+15+12+7 = 50 ✓

Inscribed: 15+12+7+0 = 34 ✓

Average: 84/2 = 42 ms ✓

other numbers can be used (rectangles)

Must average lower + upper limits for full marks

End of questions

Additional working space

Question number: _____

Additional working space

Question number: _____

Additional working space

Question number: _____

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